



CATIA V5 Training
Folds

Introduction to the Mathematical Concepts of CATIA V5

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Instructor Notes:

About this course

Objectives of the course

Upon completion of this course you will be able to:

- Understand the mathematical concepts for curve and surface definition in CATIA V5.

Targeted audience

GSD and/or FreeStyle users

Prerequisites

Students attending this course must have knowledge of GSD and FreeStyle Fundamentals



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Instructor Notes:

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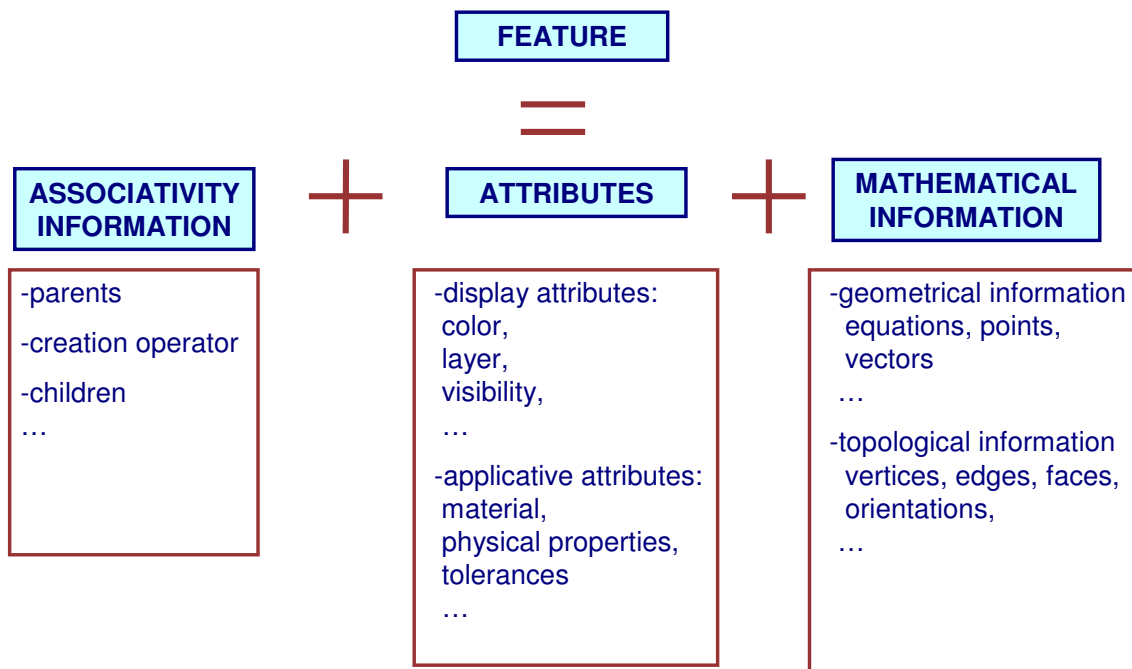
The Feature Approach (1/2)

- CATIA V5 supports a **FEATURE APPROACH**.
- It means that users create and handle objects which are **more than mathematical objects** because they carry more than just mathematical definitions.
- The mathematical definition of the object is no more than one of the **representations of the feature** which CATIA may refer to when needed.
 - ◆ For example, another representation of a surface in CATIA V5 is its triangular mesh used for shaded display or draft analysis.

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Instructor Notes:

The Feature Approach (2/2)



Note: a Datum is a feature with no parents nor creation operator in the associativity information (it may only have children)

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Instructor Notes:

The Mathematical Level

- The mathematical part of the object definition includes both **geometry** and **topology**.
 - ◆ The geometry defines the shape itself and its location in space,
 - ◆ The geometry is defined by mathematical objects such as points, vectors, angles, polynomials, ...
 - ◆ The topology ensures the consistent assembly of the geometrical elements (connections, orientations)
 - ◆ It is defined by mathematical objects such as vertices, edges, faces

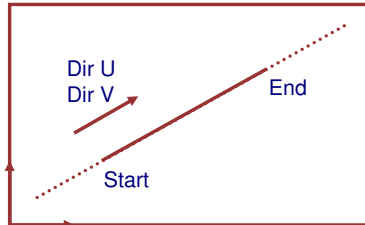
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Instructor Notes:

The Geometry Level for Curves (1/6)

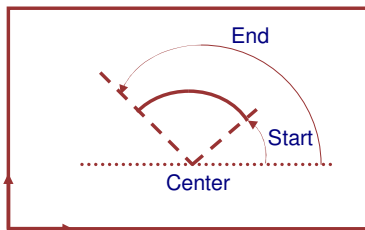
- Curves are described by canonic or parametric forms

- Examples of canonic forms



Line defined by:

- An underlying surface (may be a plane)
- An origin point
- A direction on underlying surface
- A start position
- An end position



Circle defined by:

- underlying surface
- center
- radius
- start angle
- end angle

Canonic forms

- are compact (little data)
- are exact
- make it easy to benefit from the characteristics of the object (example: select a line to define a direction)

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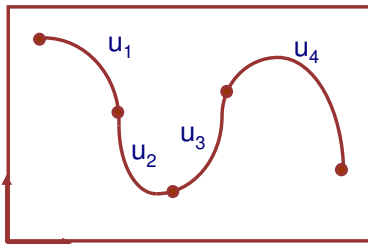
Instructor Notes:

The Geometry Level for Curves (2/6)

- Other types of curves: parametric curves = NURBS

NURBS = Non-Uniform Rational B-Spline

Non-Uniform



A NURBS curve may be described by several arcs, or **spans**, or **segments**. Each segment is described by a parametric form: it has its own set of parametric representations, for example segment number i :

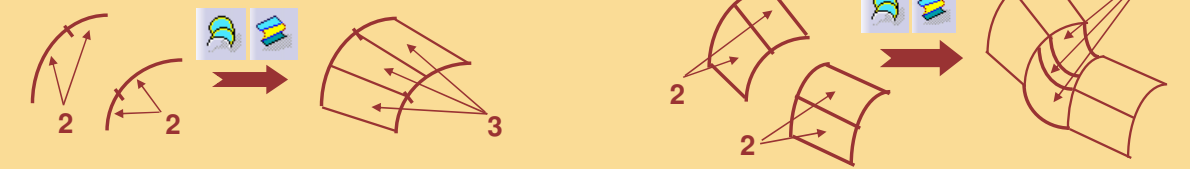
$$X = F_{xi}(u_i)$$

$$Y = F_{yi}(u_i)$$

$$Z = F_{zi}(u_i)$$

Note: the segments cannot be separated by the Disassemble command.

- + possibility to describe more complex shapes with single objects
- segmentation tends to explode if not controlled



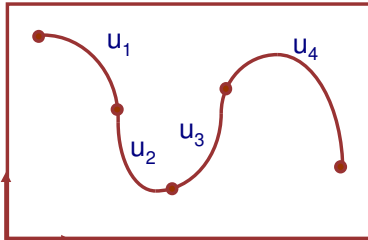
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Instructor Notes:

The Geometry Level for Curves (3/6)

NURBS = Non-Uniform Rational B-Spline

◆ Rational

Each segment is described by a rational form $X = F_{xi}(u_i) = \frac{P_{xi}(u_i)}{Q_{xi}(u_i)}$ 

With $P_{xi}(u_i)$ and $Q_{xi}(u_i)$ being polynomials,
i.e. mathematical forms such as:

$$P_{xi}(u) = A_0 + A_1 \cdot u + A_2 \cdot u^2 + \dots + A_n \cdot u^n$$

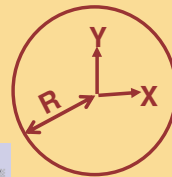
NURBS created in CATIA V5 are usually polynomial, $Q(u) = 1$
this is why they are called NUPBS (P for Polynomial)

+ possibility to describe exact conics, for example a circle can be given by:

$$X = R \frac{1 - u^2}{1 + u^2} \quad Y = R \frac{2u}{1 + u^2} \quad \dots \text{but canonic forms are also exact}$$

- degrees tend to explode if not controlled

example: ruled surface on two curves given by $\frac{P_1(u)}{Q_1(u)}$ and $\frac{P_2(u)}{Q_2(u)}$



$$S(u,v) = (1-v) \frac{P_1(u)}{Q_1(u)} + v \frac{P_2(u)}{Q_2(u)} = \frac{(1-v) P_1(u) Q_2(u) + v P_2(u) Q_1(u)}{Q_1(u) Q_2(u)} \quad \text{degrees of polynomials add}$$

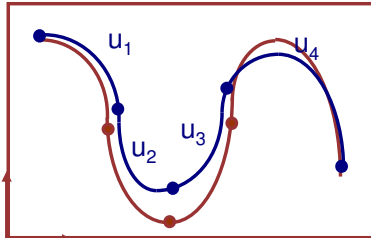
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Instructor Notes:

The Geometry Level for Curves (4/6)

NURBS = Non-Uniform Rational B-Spline

◆ B-Splines

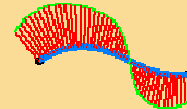


The definition of a B-Spline curves includes the description of the transitions between its segments.

Note: in CATIA V5, NURBS are always internally curvature continuous,
= transitions between segments are always C2.

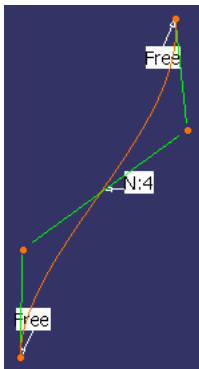
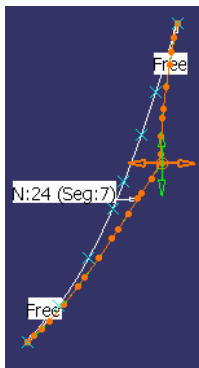
+ possibility to safely manipulate complex objects, for example to deform complex curves while preserving their overall smoothness (no unexpected gap or sharp corner appearing)

- It may be difficult manipulate the curve while keeping it good looking (example: deform by control points while keeping a nice curvature distribution)



Instructor Notes:

The Geometry Level for Curves (5/6)



Note 1: the polygonal representation

- A NURBS can be represented by a polygon = a set of **control points**
- This representation is often used in style design for intuitive shaping

Note 2: a special case of NURBS

- A NURBS can be uniform (only one segment)
- It can also be polynomial ($Q_{xi}(u_i) = 1$)

This type of curves is known as **Bezier curve**

It is favored by style designers because it is easier to manipulate (fewer points, well known properties)

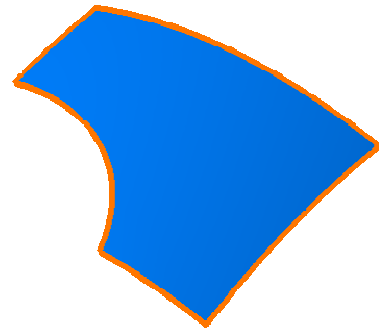
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Instructor Notes:

The Geometry Level for Curves (6/6)

General validity criteria for curves:

- A mono-cell curve must be **C2 continuous**, i.e. mathematically curvature continuous.
- It means that if an action produces a curve that is not C2 continuous, it is cut at each discontinuity and the C2 pieces become **cells** which are called **edges** and are **assembled in a topology**.
- The topology consists in a list of edges with shared vertices (common to several edges) and free vertices (common to one edge only = end points).
 - ◆ Example: the boundary feature is a single CATIA curve which is not C2 continuous.
 - ◆ This CATIA curve is made of several C2 continuous curves called edges that are assembled by a topology (joined).
 - ◆ The edges may be isolated from each other by an Extract or a Disassemble command (option All Cells).



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Instructor Notes:

The Geometry Level for Surfaces (1/5)

- Surfaces can also be described by canonic or parametric forms
- Some surfaces can also be described by their creation process
- Since R14 CATIA V5 also handles subdivision surfaces
- Examples of canonical surfaces:

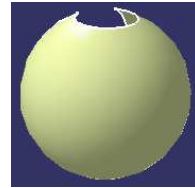
◆ Plane



◆ Cone



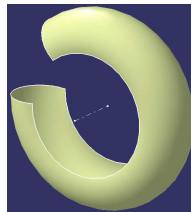
◆ Sphere



◆ Cylinder



◆ Torus



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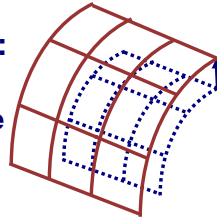
The Geometry Level for Surfaces (2/5)

Procedural surfaces

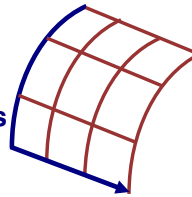
- ◆ A procedural surface is described by a creation process and the corresponding input

Examples of procedural surfaces:

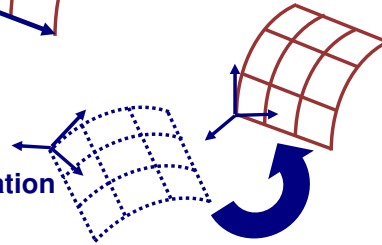
- ◆ Offset surface
defined by a surface + a distance



- ◆ Tabulated cylinder
defined by a curve, a direction, two lengths



- ◆ Linear transformation surface
defined by a surface and a geometric transformation

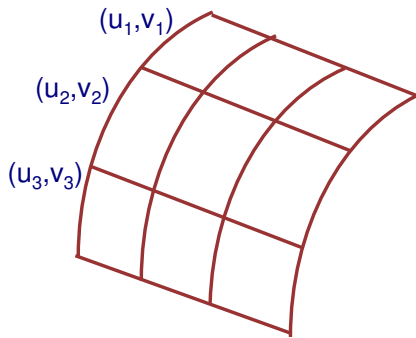


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Instructor Notes:

The Geometry Level for Surfaces (3/5)

Parametric surfaces = NURBS



The definition for surfaces is similar to the definition for curves with 2 parameters: surfaces may be described by several segments (**Non Uniform**), each segment is described by a rational form (**Rational**), but surfaces can be handled globally thanks to **B-Spline** techniques.

$$\begin{aligned} X &= F_{Xi}(u_i, v_i) = \frac{P_{xi}(u_i, v_i)}{Q_{xi}(u_i, v_i)} \\ Y &= F_{Yi}(u_i, v_i) \\ Z &= F_{Zi}(u_i, v_i) \end{aligned}$$

Notes:

- NURBS created in CATIA V5 are usually (almost always) polynomials (NUPS)
- they are always curvature continuous (C2),
- NURBS surfaces can be represented and handled by control points,
- uniform polynomial NURBS are known as Bezier patches

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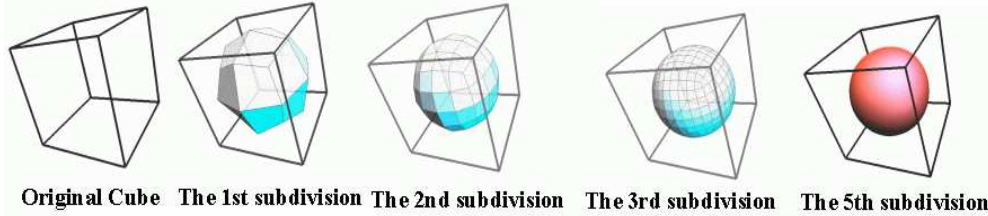
Instructor Notes:

The Geometry Level for Surfaces (4/5)

Imagine
& Shape

Subdivision surfaces

Subdivision is an algorithmic technique to generate smooth surfaces as a sequence of successively refined polyhedral meshes.

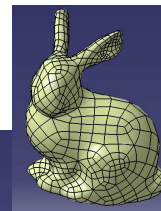
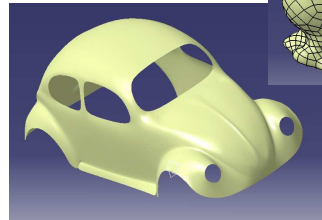
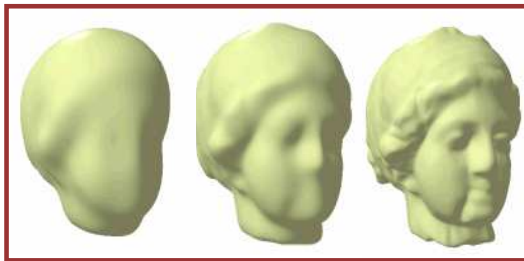


Advantages:

A complex object can be represented with only one multi-faced surface

The surface is refined only where required (details)

=> Easy manipulation + Data size reduced



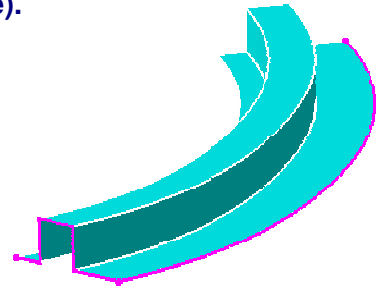
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The Geometry Level for Surfaces (5/5)

General validity criteria for surfaces:

- ❖ A mono-cell surface must be **C2 continuous**.
- ❖ It means that if an action produces a result that is not C2 continuous, it is cut at each discontinuity and the C2 pieces become **cells** which are called **faces** and are **assembled in a topology**.
- ❖ Example: the sweep feature is a single surface which is not C2 continuous (not even C1 in this case). It is made of several C2 continuous faces that are assembled by a topology (joined). The geometric surfaces may be isolated from each other by a Disassemble command (option All Cells).



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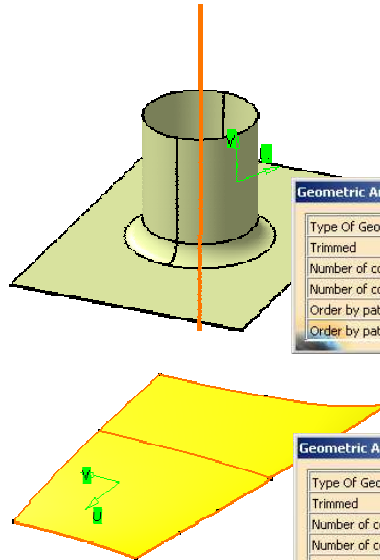
Object Analysis

- FreeStyle offers tools to analyze objects.

Dress-up options: applicable to NURBS curves and surfaces only



Geometry analysis: applicable to all curves and surfaces



| Geometric Analysis | |
|-------------------------|----------|
| Type Of Geometry | Cylinder |
| Trimmed | Yes |
| Number of components U | - |
| Number of components V | - |
| Order by patch/arc in U | - |
| Order by patch/arc in V | - |

| Geometric Analysis | |
|-------------------------|--------------|
| Type Of Geometry | NurbsSurface |
| Trimmed | Yes |
| Number of components U | 1 |
| Number of components V | 3 |
| Order by patch/arc in U | 6 |
| Order by patch/arc in V | 5 |

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